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Charnley AK (1992). Mechanisms of fungal pathogenesis in insects with particular reference to locusts. In: Lomer CJ, Prior C (eds) Biological Controls of Locusts and Grasshoppers: Proceedings of an international workshop held at Cotonou, Benin. Oxford: CAB International, pp 181-190.

Mundree SG, Farrant JM (2000). Some physiological and molecular insights into the mechanisms of desiccation tolerance in the resurrection plant Xerophyta viscasa Baker. In Cherry et al. (eds) Plant tolerance to abiotic stresses in Agriculture: Role of Genetic Engineering, Kluwer Academic Publishers, Netherlands, pp 201-222.

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International Journal of Physical Sciences

Full Length Research Paper

Numerical solution for a class of singular integral equations

Samah M. Dardery[1]* and Mohamed M. Allan[2]

[1]Department of Mathematics, Faculty of Science, Zagazig University, Egypt.
[2]Department of Mathematics, Faculty of Science and Arts Al-Mithnab, Qassim University, KSA, Saudi Arabia.

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This paper is concerned with finding approximate solution for the singular integral equations. Relating the singular integrals to Cauchy principal-value integrals, we expand the kernel and the density function of singular integral equation by the sum of the chebyshev polynomials of the first, second, third and fourth kinds. Some numerical examples are presented to illustrate the accuracy and effectiveness of the present work. Numerical results show that the errors of approximate solutions of examples in different cases with small value of n are very small. These show that the methods developed are very accurate and in fact for a linear function give the exact solution.

Key words: Singular integral equations, Cauchy kernel, chebyshev polynomials, weight functions.

INTRODUCTION

During the last three decades, the singular integral equation methods with applications to several basic fields of engineering mechanics, like elasticity, plasticity, aerodynamics and fracture mechanics have been studied and improved by several scientists (Chakrabarti, 1989; Ladopoulous, 2000, 1987; Zabreyko, 1975; Prossdorf, 1977; Zisis and Ladopoulos, 1989). Hence, it is of interest to solve numerically this type of integral equations (Chakrabarti and Berghe, 2004; Abdou and Naser, 2003). Chebyshev polynomials are of great importance in many areas of mathematics particularly approximation theory (Abdulkawi et al., 2009; Eshkuvatov et al., 2009).

In this paper, we analyze the numerical solution of singular integral equations by using Chebyshev polynomials of first, second, third and fourth kind to obtain systems of linear algebraic equations; these systems are solved numerically. The methodology of the present work is expected to be useful for solving singular integral equations of the first kind, involving partly singular and partly regular kernels. The singularity is assumed to be of the Cauchy type. The method is illustrated by considering some examples.

Singular integral equation of first kind, with a Cauchy type singular kernel, over a finite interval can be represented by:

$$\int_{-1}^{1}\frac{k(t,x)\varphi(t)}{t-x}dt+\int_{-1}^{1}L(t,x)\varphi(t)\,dt=f(x),\qquad -1<x<1 \tag{1}$$

where $k(t,x), L(t,x)$ and $f(x)$ are given real-valued continuous functions belonging to the class Holder of continues functions and $k(t,t)\neq 0$. In Equation (1) the singular kernel is interpreted as Cauchy principle value. Integral equation of form 1 and other different forms have many applications (Chakrabarti, 1989; Ladopoulous, 2000; Ladopoulous, 1987; Gakhov, 1966; Martin and Rizzo, 1989; Zisis and Ladopoulos, 1989). The theory of this equation is well known and it is presented in Sheshko (2003) and Muskhelishvili (1977). An approximate method for solving Equation (1) using a polynomial approximation of degree n has been proposed by Chakrabarti and Berghe (2004).

*Corresponding author. E-mail: sd.1974@hotmail.com

It is well known that the analytical solutions of the simple singular integral equation

$$\int_{-1}^{1} \frac{\varphi(t)}{t-x} dt = f(x), \qquad -1 < x < 1 \tag{2}$$

at $k(t,x) = 1$ and $L(t,x) = 0$, for the following four cases,

(I) The solution is unbounded at both end-points $x = \pm 1$,

(II) The solution is bounded at both end-points $x = \pm 1$,

(III) The solution is bounded at end $x = -1$, but unbounded at end $x = +1$,

(IV) The solution is unbounded at end $x = -1$, but bounded at end $x = +1$,

are given by Lifanov (1996). In this paper, the used approximate method for solving Equation 1 stems from recent work (Eshkuvatov et al., 2009) wherein an approximate method has been developed to solve the simple Equation (2). The approximate method developed below appears to be quite appropriate for solving the most general type Equations (1). Some examples are presented to illustrate the method.

THE APPROXIMATE SOLUTION

In this section, we present the method of the approximate solution of Equation (1) in four cases. Let the unknown function φ in Equation (1) be approximated by the polynomial function

$$\varphi_n(x) = W^{(j)}(x) \sum_{i=0}^{n} c_i^{(j)} \Psi_i^{(j)}(x) \qquad (j = 1,2,3,4) \tag{3}$$

Where $c_i^{(j)},\ i = 0,1,2,...,n$ are unknown coefficients and in case (I): $\Psi_i^{(1)}(x) = T_i(x)$, in case (II): $\Psi_i^{(2)}(x) = U_i(x)$, in case (III): $\Psi_i^{(3)}(x) = V_i(x)$ and in case (VI): $\Psi_i^{(4)}(x) = W_i(x)$, where T_i, U_i, V_i and $W_i,\ i = 0,1,...n,$ are the Chebyshev polynomials of the first, second, third and fourth kinds respectively can be defined by the recurrence relations (Prem and Michael, 2005; Abdulkawi et al., 2009).

$$\left. \begin{array}{ll} T_0(x) = 1, & T_1(x) = x \\ T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x) & n \geq 2 \end{array} \right\} \tag{4}$$

$$\left. \begin{array}{ll} U_0(x) = 1, & U_1(x) = 2x \\ U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x) & n \geq 2 \end{array} \right\} \tag{5}$$

$$\left. \begin{array}{ll} V_0(x) = 1, & V_1(x) = 2x - 1 \\ V_n(x) = 2xV_{n-1}(x) - V_{n-2}(x) & n \geq 2 \end{array} \right\} \tag{6}$$

$$\left. \begin{array}{ll} W_0(x) = 1, & W_1(x) = 2x + 1 \\ W_n(x) = 2xW_{n-1}(x) - W_{n-2}(x) & n \geq 2 \end{array} \right\} \tag{7}$$

and $W^i,\ i = 0,1,...n,$ are the corresponding weight functions. Substituting the approximate solution Equation (3) for the unknown function into Equation (1) yields:

$$\sum_{i=0}^{n} c_i^{(j)} \left[\int_{-1}^{1} \frac{k(t,x) W^{(j)}(t) \Psi_i^{(j)}(t)}{t-x} dt + \int_{-1}^{1} L(t,x) W^{(j)}(t) \Psi_i^{(j)}(t)\, dt \right] = f(x), \ -1 < x < 1 \tag{8}$$

In above Equation (8), we next use the following chebyshev approximation to the kernels $k(t,x)$ and $L(t,x)$, given by (for fixed x, cf.(Chakrabarti and Berghe, 2004)

$$k(t,x) \cong \sum_{p=0}^{m} k_p(x) t^p \ , \qquad L(t,x) \cong \sum_{q=0}^{s} L_q(x) t^q \tag{9}$$

with known expressions for $K_p(x)$ and $L_q(x)$. Then Equation 8 gives

$$\sum_{i=0}^{n} c_i^{(j)} \alpha_i^{(j)}(x) = f(x), \qquad -1 < x < 1,\ (j = 1,2,3,4) \tag{10}$$

Where

$$\alpha_i^{(j)}(x) = \sum_{p=0}^{m} k_p(x) u_{p,i}^{(j)}(x) + \sum_{q=0}^{s} L_q(x) \gamma_{q,i}^{(j)} \tag{11}$$

With

$$u_{p,i}^{(j)}(x) = \int_{-1}^{1} \frac{t^p W^{(j)}(t) \Psi_i^{(j)}(t)}{t-x} dt \qquad -1 < x < 1, \quad (j = 1,2,3,4) \tag{12}$$

And

$$\gamma_{q,i}^{(j)} = \int_{-1}^{1} t^q W^{(j)}(t) \Psi_i^{(j)}(t)\, dt \tag{13}$$

Let $x_k^{(j)},\ j = 1,2,3,4$, be the zeros of $U_n(x), T_{n+2}(x), W_{n+1}(x)$ and $V_{n+1}(x)$, respectively.

Substituting the collocation points $x_k^{(j)},\ j=1,2,3,4$ into Equation 10, we obtain the following systems of linear equations:

$$\sum_{i=0}^{n} c_i^{(j)} \alpha_i^{(j)}(x_k^{(j)}) = f(x_k^{(j)}), \quad (k=1,2,\ldots,n+1), \ (j=1,2,3,4) \tag{14}$$

where

$$\alpha_i^{(j)}(x_k^{(j)}) = \sum_{p=0}^{m} k_p(x_k^{(j)}) u_{p,i}^{(j)}(x_k^{(j)}) + \sum_{q=0}^{s} L_q(x_k^{(j)}) \gamma_{q,i}^{(j)}, \ (k=1,2,\ldots,n+1), (j=1,2,3,4) \tag{15}$$

Solving the system of Equation 14 for the unknown coefficients $c_i^{(j)},\ j=1,2,3,4$, and substituting the values of $c_i^{(j)}$ into Equation 3 we obtain the approximate solutions of Equation 1 in the form of

$$\varphi_n(x) \cong W^{(j)}(x) \sum_{i=0}^{n} c_i^{(j)} \Psi_i^{(j)}(x) \qquad (j=1,2,3,4) \tag{16}$$

NUMERICAL EXAMPLES

In this section, we consider some problems to illustrate the above method. All results were computed using FORTRAN code.

Example 1. Consider the following singular integral equation

$$\int_{-1}^{1} \frac{(x+t^2)\varphi(t)}{t-x} + \int_{-1}^{1} (x^2+t^3)\varphi(t)\,dt = 2x^4 - 2x^2 - \frac{3}{8}, \qquad -1<x<1 \tag{17}$$

Where

$k(x,t) = x+t^2, \quad L(x,t) = x^2+t^3, \quad f(t) = 2x^4 - 2x^2 - \frac{3}{8}$.

So, one gets

$$k_0(x) = x,\ k_1(x) = 0,\ k_2(x) = 1 \quad k_p(x) = 0, \qquad (p>2)$$
$$L_0(x) = x^2,\ L_1(x) = 0,\ L_2(x) = 0,\ L_3(x) = 1,\ L_q(x) = 0 \quad (q>3)$$

Hence we find that relation (10) produces

$$\sum_{i=0}^{n} c_i^{(j)} \alpha_i^{(j)}(x) = 2x^4 - 2x^2 - \frac{3}{8} \qquad -1<x<1, \ (j=1,2,3,4)$$

Thus Equation 11 gives

$$\alpha_i^{(j)}(x) = x u_{0,i}^{(j)}(x) + u_{2,i}^{(j)}(x) + x^2 \gamma_{0,i}^{(j)} + \gamma_{3,i}^{(j)}, \ (j=1,2,3,4), \ (i=0,1,2,\ldots)$$

Firstly, let us consider in detail the case (I) , $j=1$, for $n=3$. This result in

$$u_{0,i}^{(1)}(x) = \int_{-1}^{1} \frac{T_i(t)}{\sqrt{1-t^2}\, t-x} dt, \quad u_{2,i}^{(1)}(x) = \int_{-1}^{1} \frac{t^2 T_i(t)}{\sqrt{1-t^2}\, t-x} dt, \ -1<t<1, \tag{18}$$

$$\gamma_{0,i}^{(1)} = \int_{-1}^{1} \frac{T_i(t)}{\sqrt{1-t^2}} dt, \qquad \gamma_{3,i}^{(1)} = \int_{-1}^{1} \frac{t^3 T_i(t)}{\sqrt{1-t^2}} dt, \tag{19}$$

By applying the following relations

$$\int_{-1}^{1} \frac{T_i(t)}{\sqrt{1-t^2}(t-x)} dt = \pi U_{i-1}(x), \qquad \int_{-1}^{1} \frac{1}{\sqrt{1-t^2}(t-x)} dt = 0 \tag{20}$$

$$\int_{-1}^{1} \frac{T_i(t) T_j(t)}{\sqrt{1-t^2}} dt = \begin{cases} 0 & i \neq j \\ \pi & i = j = 0 \\ \pi/2 & i = j \neq 0 \end{cases} \tag{21}$$

It is easy to estimate the values $u_{0,i}^{(1)}, u_{2,i}^{(1)}, \gamma_{0,i}^{(1)}$ and $\gamma_{3,i}^{(1)}$. From Equations 10 and 18-21 we get

$$\alpha_i^{(1)}(x) = \begin{cases} \pi(x^2+x); & i=0 \\ \pi\left(x^2+x+\frac{7}{8}\right); & i=1 \\ \pi(2x^3+2x^2); & i=2 \\ \pi\left(4x^4+4x^3-x^2-x+\frac{1}{8}\right); & i=3 \end{cases} \tag{22}$$

By choosing the collocation points $x_k = \cos\left(\frac{(2k-1)\pi}{2(n+2)}\right), (k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3} c_i^{(1)} \alpha_i^{(1)}(x_k) = f(x_k), \quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(1)},\ i=0,1,2,3$ that produces

$$\left.\begin{matrix} c_0^{(1)} = 0.3183098, & c_1^{(1)} = -0.1591549 \\ c_2^{(1)} = -0.3183098, & c_3^{(1)} = 0.1591549 \end{matrix}\right\} \tag{23}$$

From Equation (23) we obtain the approximate solution of Equation (17) in the form

$$\varphi_n(x) \cong \frac{2}{\pi\sqrt{1-x^2}}(x^3 - x^2 - x + 1) \tag{24}$$

Which coincides with the exact solution. The error of approximate solution (24) of Equation 17 at $n=20$ is given in Table 1.

Table 1. Errors of approximate solutions of Equation 17 in Cases (I)-(IV) at *n*=20.

X	error (j=1)	error (j=2)	error (j=3)	error (j=4)
-9.500000E-01	0.000000E+00	0.000000E+00	5.960464E-08	5.960464E-08
-9.000000E-01	0.000000E+00	0.000000E+00	1.192093E-07	1.192093E-07
-7.000000E-01	0.000000E+00	0.000000E+00	1.192093E-07	1.192093E-07
-5.000000E-01	5.960464E-08	5.960464E-08	1.788139E-07	1.788139E-07
-3.000000E-01	0.000000E+00	5.960464E-08	1.788139E-07	1.788139E-07
-1.000000E-01	5.960464E-08	5.960464E-08	1.192093E-07	1.192093E-07
0.000000E+00	5.960464E-08	5.960464E-08	1.192093E-07	1.192093E-07
1.000000E-01	1.192093E-07	5.960464E-08	5.960464E-08	5.960464E-08
3.000000E-01	8.940697E-08	8.940697E-08	8.940697E-08	8.940697E-08
5.000000E-01	8.940697E-08	8.940697E-08	5.960464E-08	5.960464E-08
7.000000E-01	1.043081E-07	7.450581E-08	1.490116E-08	1.490116E-08
9.000000E-01	9.313226E-08	5.029142E-08	4.656613E-08	1.303852E-08
9.500000E-01	5.774200E-08	3.632158E-08	6.705523E-08	4.656613E-09

Secondly, let us consider in detail the case (II) , $j=2$, for $n=3$. This result in

$$u_{0,i}^{(2)}(x)=\int_{-1}^{1}\frac{\sqrt{1-t^2}\,U_i(t)}{t-x}dt,\qquad u_{2,i}^{(2)}(x)=\int_{-1}^{1}\frac{t^2\sqrt{1-t^2}\,U_i(t)}{t-x}dt,\quad -1<t<1, \tag{25}$$

$$\gamma_{0,i}^{(2)}=\int_{-1}^{1}\sqrt{1-t^2}U_i(t)\,dt,\qquad \gamma_{3,i}^{(2)}=\int_{-1}^{1}\sqrt{1-t^2}\;t^3U_i(t)\,dt, \tag{26}$$

By applying the following relations

$$\int_{-1}^{1}\frac{\sqrt{1-t^2}U_i(t)}{t-x}dt=-\pi T_{i+1}(x) \tag{27}$$

$$\int_{-1}^{1}\sqrt{1-t^2}U_i(t)U_j(t)\;dt=\begin{cases}0 & i\neq j\\ \dfrac{\pi}{2} & i=j\end{cases} \tag{28}$$

It is easy to estimate the values $u_{0,i}^{(2)},u_{2,i}^{(2)},\gamma_{0,i}^{(2)}$ and $\gamma_{3,i}^{(2)}$. From the relations 11 and Equations (25) to (28) we get

$$\alpha_i^{(2)}(x)=\begin{cases}-\dfrac{\pi}{2}\left(2x^3+x^2-x\right) & i=0\\ -\pi\left(2x^4+2x^3-x^2-x-\dfrac{3}{8}\right) & i=1\\ -\pi\left(4x^5+4x^4-3x^3-3x^2+\dfrac{1}{4}x\right) & i=2\\ -\pi\left(8x^6+8x^5-8x^4-8x^3+x^2+x-\dfrac{1}{16}\right) & i=3\end{cases} \tag{29}$$

By choosing the collocation points $x_k^{(2)}=\cos\left(\dfrac{(2k-1)\pi}{2(n+2)}\right),(k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3}c_i^{(2)}\,\alpha_i^{(2)}(x_k^{(2)})=f(x_k^{(2)}),\quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(2)},\ i=0,1,2,3$ that produces

$$\left.\begin{array}{ll}c_0^{(2)}=0.6366197, & c_1^{(2)}=-0.3183099\\ c_2^{(2)}=2.279989\times10^{-8}, & c_3^{(2)}=-7.819254\times10^{-9}\end{array}\right\} \tag{30}$$

From Equation (30) we obtain the approximate solution of Equation (17) in the form

$$\varphi_n(x)\cong\frac{2\sqrt{1-x^2}}{\pi}(1-x) \tag{31}$$

Which coincides with the exact solution. The error of approximate Solution (31) of Equation (17) at $n=20$ is given in Table 1.

Thirdly, let us consider in detail the case (III) , $j=3$, for $n=3$. This result in

$$u_{0,i}^{(3)}(x)=\int_{-1}^{1}\sqrt{\frac{1+t}{1-t}}\frac{V_i(t)}{t-x}dt,\qquad u_{2,i}^{(3)}(x)=\int_{-1}^{1}\sqrt{\frac{1+t}{1-t}}\;\frac{t^2V_i(t)}{t-x}dt,\quad -1<t<1, \tag{32}$$

$$\gamma_{0,i}^{(3)}=\int_{-1}^{1}\sqrt{\frac{1+t}{1-t}}V_i(t)\,dt,\qquad \gamma_{3,i}^{(3)}=\int_{-1}^{1}\sqrt{\frac{1+t}{1-t}}\;t^3V_i(t)\,dt, \tag{33}$$

By applying the following relations

$$\int_{-1}^{1}\sqrt{\frac{1+t}{1-t}}V_i(t)V_j(t)dt=\begin{cases}0 & i\neq j\\ \pi & i=j\end{cases} \tag{34}$$

$$\int_{-1}^{1}\sqrt{\frac{1+t}{1-t}}\frac{V_i(t)}{t-x}dt=\pi W_i(x) \tag{35}$$

It is easy to estimate the values $u_{0,i}^{(3)}, u_{2,i}^{(3)}, \gamma_{0,i}^{(3)}$ and $\gamma_{3,i}^{(3)}$. From the relations (6) and Equations (32) to (35) we get

$$\alpha_i^{(3)}(t)=\begin{cases}\pi\left(2x^2+2x+\frac{7}{8}\right) & i=0\\ \pi\left(2x^3+3x^2+x+\frac{7}{8}\right) & i=1\\ \pi\left(4x^4+6x^3+x^2-x+\frac{1}{8}\right) & i=2\\ \pi\left(8x^5+12x^4-5x^2+x+\frac{1}{8}\right) & i=3\end{cases} \tag{36}$$

By choosing the collocation points $x_k^{(3)}=\cos\left(\frac{2k\pi}{(2n+3)}\right),(k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3}c_i^{(3)}\,\alpha_i^{(3)}(x_k^{(3)})=f(x_k^{(3)}),\quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(3)},\ i=0,1,2,3$ that produces

$$\left.\begin{matrix}c_0^{(3)}=0.3183098, & c_1^{(3)}=-0.4774647,\\ c_2^{(3)}=0.1591549, & c_3^{(3)}=1.330901\times10^{-8}\end{matrix}\right\} \tag{37}$$

From Equation (37) we obtain the approximate solution of Equation (17) in the form of

$$\varphi_n(x)\cong\frac{2}{\pi}\sqrt{\frac{1+x}{1-x}}\left(x^2-2x+1\right) \tag{38}$$

Which coincides with the exact solution. The error of approximate Solution (38) of Equation (17) at $n=20$ is given in Table 1.

Fourthly, In case (IV), $j=4$, for n=3. This result in

$$u_{0,i}^{(4)}(x)=\int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}\frac{W_i(t)}{t-x}dt,\quad u_{2,i}^{(4)}(x)=\int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}\frac{t^2W_i(t)}{t-x}dt,\quad -1<t<1, \tag{39}$$

$$\gamma_{0,i}^{(4)}=\int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}W_i(t)\,dt,\qquad \gamma_{3,i}^{(4)}=\int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}\,t^3W_i(t)\,dt, \tag{40}$$

By applying the relations

$$\int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}W_i(t)W_j(t)dt=\begin{cases}0 & i\neq j\\ \pi & i=j\end{cases} \tag{41}$$

$$\int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}\frac{W_i(t)}{t-x}dt=-\pi V_i(x) \tag{42}$$

It is easy to estimate the values $u_{0,i}^{(4)}, u_{2,i}^{(4)}, \gamma_{0,i}^{(4)}$ and $\gamma_{3,i}^{(4)}$. From the relations (7) and Equations (39) to (42) we get

$$\alpha_i^{(4)}(x)=\begin{cases}\frac{-7\pi}{8}; & i=0\\ -\pi\left(2x^3+x^2-x-\frac{7}{8}\right); & i=1\\ -\pi\left(4x^4+2x^3-3x^2-x+\frac{1}{8}\right); & i=2\\ -\pi\left(8x^5+4x^4-8x^3-3x^2+x-\frac{1}{8}\right); & i=3\end{cases} \tag{43}$$

By choosing the collocation points $x_k^{(4)}=\cos\left(\frac{(2k-1)\pi}{(2n+3)}\right),(k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3}c_i^{(4)}\,\alpha_i^{(4)}(x_k^{(4)})=f(x_k^{(4)}),\quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(4)},\ i=0,1,2,3$ that produces

$$\left.\begin{matrix}c_0^{(4)}=0.3183098, & c_1^{(4)}=0.1591549\\ c_2^{(4)}=-0.1591549, & c_3^{(4)}=2.358931\times10^{-8}\end{matrix}\right\} \tag{44}$$

From Equation (44) we obtain the approximate solution of Equation (17) in the form of

$$\varphi_n(x)\cong\frac{-2}{\pi}\sqrt{\frac{1-x}{1+x}}(x^2-1) \tag{45}$$

Which coincides with the exact solution. The error of approximate Solution (45) of Equation 46 at $n=20$ is given in Table 1.

Example2 . Consider the following singular integral equation

$$\int_{-1}^{1}\frac{\varphi(t)}{t-x}dt+\int_{-1}^{1}(x^3+xt^2)\varphi(t)\,dt=x^3+x \qquad (46)$$

Which corresponds with $k(t,x)=1$ and $L(t,x)=x^3+xt^2$. So one gets

$$k_0(x)=1,\quad k_p(x)=0,\qquad (p>0)$$
$$L_0(x)=x^3,\; L_1(x)=0,\; L_2(x)=x,\; L_q(x)=0\quad (q>2)$$

Hence we find that relation (10) produces

$$\sum_{i=0}^{n}c_i^{(j)}\alpha_i^{(j)}(x)=x^3+x \qquad -1<x<1,\ (j=1,2,3,4)$$

Thus (11) gives

$$\alpha_i^{(j)}(x)=u_{0,i}^{(j)}(x)+x^3\gamma_{0,i}^{(j)}+x\gamma_{2,i}^{(j)},\ (j=1,2,3,4),\ (i=0,1,2,...)$$

Firstly, let us consider in detail the case (I) , $j=1$, for $n=3$. This result in

$$\gamma_{2,i}^{(1)}=\int_{-1}^{1}\frac{t^2T_i(t)}{\sqrt{1-t^2}}dt, \qquad (47)$$

From the relations (18) to (21) and (47) we obtain

$$\alpha_i^{(1)}(x)=\begin{cases}\pi\left(x^3+x/2\right) & i=0\\ \pi & i=1\\ 9\pi x/4 & i=2\\ \pi\left(4x^2-1\right) & i=3\end{cases} \qquad (48)$$

By choosing the collocation points $x_k=\cos\left(\frac{(2k-1)\pi}{2(n+2)}\right),(k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3}c_i^{(1)}\,\alpha_i^{(1)}(x_k)=f(x_k),\quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(1)},\ i=0,1,2,3$ that produces

$$\left.\begin{array}{ll}c_0^{(1)}=0.3183098, & c_1^{(1)}=1.090772\times10^{-8}\\ c_2^{(1)}=0.07073557, & c_3^{(1)}=1.830649\times10^{-8}\end{array}\right\} \qquad (49)$$

From (49) we obtain the approximate solution of Equation 46 in the form of

$$\varphi_n(x)\cong\frac{1}{9\pi\sqrt{1-x^2}}\left(7+4x^2\right) \qquad (50)$$

Which coincides with the exact solution. The error of approximate Solution (50) of Equation (46) at $n=20$ is given in Table 2.

Secondly, let us consider in detail the case (II) , $j=2$, for $n=3$. This result in

$$\gamma_{2,i}^{(2)}=\int_{-1}^{1}\sqrt{1-t^2}\,t^2U_i(t)\,dt, \qquad (51)$$

By applying the relations (25)-(28) and (51) we get

$$\alpha_i^{(2)}(x)=\begin{cases}\frac{\pi}{2}\left(x^3-\frac{7x}{4}\right) & i=0\\ -\pi\left(2x^2-1\right) & i=1\\ -\pi\left(4x^3-\frac{25x}{8}\right) & i=2\\ -\pi\left(8x^4-8x^2+1\right) & i=3\end{cases} \qquad (52)$$

By choosing the collocation points $x_k^{(2)}=\cos\left(\frac{(2k-1)\pi}{2(n+2)}\right),(k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3}c_i^{(2)}\,\alpha_i^{(2)}(x_k^{(2)})=f(x_k^{(2)}),\quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(2)},\ i=0,1,2,3$ that produces

$$\left.\begin{array}{ll}c_0^{(2)}=-1.170559, & c_1^{(2)}=-1.331665\times10^{-9}\\ c_2^{(2)}=-0.2258973, & c_3^{(2)}=-1.644008\times10^{-8}\end{array}\right\} \qquad (53)$$

Table 2. Errors of approximate solutions of Equation 46 in Case (I), Case (II) and Case (IV) respectively at *n*=20.

x	error (j=1)	error (j=2)	error (j=4)
-9.500000E-01	0.000000E+00	0.000000E+00	0.000000E+00
-9.000000E-01	5.960464E- 08	5.960464E- 08	0.000000E+00
-7.000000E-01	8.940697E- 08	1.192093E- 07	5.960464E- 08
-5.000000E-01	8.940697E- 08	1.192093E- 07	1.192093E- 07
-3.000000E-01	8.940697E- 08	1.788139E- 07	1.192093E- 07
-1.000000E-01	1.192093E- 07	1.788139E- 07	1.788139E- 07
0.000000E+00	1.043081E- 07	1.788139E- 07	1.788139E- 07
1.000000E-01	1.192093E- 07	1.788139E- 07	1.192093E- 07
3.000000E-01	8. 940697E-08	1.788139E- 07	5.960464E- 08
5.000000E-01	8.940697E- 08	1.192093E- 07	1.192093E-07
7.000000E-01	8.940697E- 08	1.192093E- 07	0.00000E+ 00
9.000000E-01	5.9604641E-08	5.960464E- 08	5.960464E-08
9.500000E-01	0.000000E+00	0.000000E+00	0.000000E+00

From Equation (53) we obtain the approximate solution of Equation (46) in the form of

$$\varphi_n(x) \cong \frac{-\sqrt{1-x^2}}{31\pi}\left[92+88x^2\right] \qquad (54)$$

which coincides with the exact solution. The error of approximate solution (54) of Equation 46 at $n=20$ is given in Table 2.

Thirdly, In case (IV), $j=4$, for n=3. This result in

$$\gamma_{2,i}^{(4)} = \int_{-1}^{1}\sqrt{\frac{1-t}{1+t}}\; t^2 W_i(t)\,dt, \qquad (55)$$

By applying the relations (39) to (42) and (55) we get

$$\alpha_i^{(4)}(x) = \begin{cases} \pi\left(x^3+\dfrac{x}{2}-1\right) & i=0 \\ -\pi\left(\dfrac{9}{4}x-1\right) & i=1 \\ -\pi\left(4x^2-\dfrac{9x}{4}-1\right) & i=2 \\ -\pi\left(8x^3-4x^2-4x+1\right) & i=3 \end{cases} \qquad (56)$$

By choosing the collocation points $x_k^{(4)} = \cos\left(\dfrac{(2k-1)\pi}{(2n+3)}\right), (k=1,2,3,4)$, for $n=3,$ we obtain the following system of linear equations:

$$\sum_{i=0}^{3} c_i^{(4)}\,\alpha_i^{(4)}(x_k^{(4)}) = f(x_k^{(4)}), \quad k=1,2,3,4$$

By solving this system for the unknown coefficients $c_i^{(4)},\ i=0,1,2,3$ that produces

$$c_0^{(4)} = c_1^{(4)} = -.5852794, \quad c_2^{(4)} = c_3^{(4)} = -0.1129487 \qquad (57)$$

From Equation (57) we obtain the approximate solution of Equation (46) in the form of

$$\varphi_n(x) \cong \frac{-1}{31\pi}\sqrt{\frac{1-x}{1+x}}(1+x)(92+88x^2) \qquad (58)$$

which coincides with the exact solution. The error of approximate Solution (58) of Equation (46) at $n=20$ is given in Table 2.

Similarly, doing the same operations as we did for Case (I), Case (II) and Case (IV), one can solve for Case (III).

Example 3. Consider the following singular integral equation

$$\int_{-1}^{1}\frac{\varphi(t)}{t-x}dt + \int_{-1}^{1}(x^2+t^2)\varphi(t)\,dt = \frac{-3}{2}x^2+2x, \qquad (59)$$

which corresponds with $k(t,x)=1$ and $L(t,x)=x^2+t^2$. So, one gets

$$k_0(x)=1, \quad k_p(x)=0, \qquad (p>0)$$
$$L_0(x)=x^2,\ L_1(x)=0,\ L_2(x)=1,\ L_q(x)=0 \ \ (q>2)$$

Table 3. Errors of approximate solutions of Equation 59 in Case (II) and Case (III) at *n*=20.

x	error (j=2)	error (j=3)
-9.500000E-01	2.980232E-08	2.980232E-08
-9.000000E-01	2.980232E-08	5.960464E-08
-7.000000E-01	0.000000E+00	5.960464E-08
-5.000000E-01	0.000000E+00	1.192093E-07
-3.000000E-01	0.000000E+00	1.192093E-07
-1.000000E-01	5.960464E-08	1.192093E-07
0.000000E+00	5.960464E-08	1.192093E-07
1.000000E-01	5.960464E-08	1.192093E-07
3.000000E-01	1.192093E-07	1.192093E-07
5.000000E-01	1.192093E-07	8.940697E-08
7.000000E-01	1.192093E-07	0.000000E+00
9.000000E-01	8.940697E-08	1.788139E-07
9.500000E-01	5.960464E-08	3.278255E-07

Hence the relation (10) produces

$$\sum_{i=0}^{n} c_i^{(j)} \alpha_i^{(j)}(x) = \frac{-3}{2} x^2 + 2x, \qquad -1 < x < 1,\ j = 1,2,3,4 \tag{60}$$

where Equation (11) gives

$$\alpha_i^{(j)}(x) = u_{0,i}^{(j)}(x) + x^2 \gamma_{0,i}^{(j)} + \gamma_{2,i}^{(j)},\ \ (j = 1,2,3,4),\ \ (i = 0,1,2,...)$$

Firstly, let us consider in detail the case (II), $j = 2$, for $n = 3$. From Equation (25) to (28) and (51) we get

$$\alpha_i^{(2)}(x) = \begin{cases} \frac{\pi}{8}\left(4x^2 - 8x + 1\right) & i = 0 \\ -\pi\left(2x^2 - 1\right) & i = 1 \\ \frac{-\pi}{8}\left(32x^3 - 24x - 1\right) & i = 2 \\ -\pi\left(8x^4 - 8x^2 + 1\right) & i = 3 \end{cases} \tag{61}$$

By solving the system (60), at the collocation points $x_k^{(2)} = \cos\left(\frac{(2k-1)\pi}{2(n+2)}\right), (k = 1,2,3,4)$, for the unknown coefficients $c_i^{(2)},\ i = 0,1,2,3$ we obtain

$$\left.\begin{aligned} c_0^{(2)} &= -0.6366197, & c_1^{(2)} &= 0.07957754 \\ c_2^{(2)} &= 1.746461 \times 10^{-8}, & c_3^{(2)} &= 1.827517 \times 10^{-8} \end{aligned}\right\} \tag{62}$$

So the approximate solution of Equation 59 is given by

$$\varphi_n(x) \cong \frac{-\sqrt{1-x^2}}{2\pi}(4 - x) \ , \tag{63}$$

Which coincides with the exact solution, the error of the approximate solution (63) of Equation 59 at $n = 20$ is given in Table 3.

Secondly, In case (III), $j = 3$, for n=3. This result in

$$\gamma_{2,i}^{(3)} = \int_{-1}^{1} \sqrt{\frac{1+t}{1-t}}\ t^2 V_i(t)\, dt, \tag{64}$$

From (32)-(35) and (64) we get

$$\alpha_i^{(3)}(x) = \begin{cases} \pi\left(x^2 + \frac{3}{2}\right) & i = 0 \\ \pi\left(2x + \frac{5}{4}\right) & i = 1 \\ \pi\left(4x^2 + 2x - \frac{3}{4}\right) & i = 2 \\ \pi\left(8x^3 + 4x^2 - 4x + 1\right) & i = 3 \end{cases} \tag{65}$$

By solving the system (60), at the collocation points $x_k^{(3)} = \cos\left(\frac{2k\pi}{(2n+3)}\right), (k = 1,2,3,4)$, for the unknown coefficients $c_i^{(3)},\ i = 0,1,2,3$ we obtain

$$\begin{aligned} c_0^{(3)} &= -0.3183099, & c_1^{(3)} &= 0.3580987, \\ c_2^{(3)} &= -0.03978872, & c_3^{(3)} &= -8.155105 \times 10^{-9} \end{aligned} \tag{66}$$

Hence, the approximate solution of Equation 59 is given by

$$\varphi_n(x) \cong \frac{-1}{2\pi}\sqrt{\frac{1+x}{1-x}}\,(x^2-5x+4) \tag{67}$$

Which coincides with the exact solution, the error of the approximate solution (67) of Equation (59) at $n=20$ is given in Table 3.

Similarly, doing the same operations as we did for Case (II) and Case (III), one can solve for Case (I) and Case (IV). Table 1 illustrates errors of approximate solutions of Equation 17 in Cases (I)-(IV) at $n=20$. Table 2 illustrates errors of approximate solutions of Equation 46 in Case (I), Case (II) and Case (IV) respectively at $n=20$. Table 3 illustrates errors of approximate solutions of Equation 59 in Case (II) and Case (III) at $n=20$.

Conclusion

Numerical results (Tables 1, 2 and 3) show that the errors of approximate solutions of Examples 1-3 in different Cases with small value of n are very small. These show that the methods developed are very accurate and in fact for a linear function give the exact solution.

REFERENCES

Abdou M, Naser A (2003). On the numerical treatment of the singular integral equation of the second kind. Appl. Math. Comput. 146:373-380.

Abdulkawi A, Eshkuvatov Z, Long N (2009). A not on the numerical solution of singular integral equations of Cauchy type. Int. J. Appl. Math. Comp. Sci. 5(2):90-93.

Chakrabarti A (1989). Solution of two singular integral equations arising in water wave problems, ZAMM 69:457-459.

Chakrabarti A, Berghe V (2004). Approximate solution of singular integral equations. Science direct, Appl. Math. Lett. 17:553-559.

Eshkuvatov ZK, Nik Long N, Abdulkawi M (2009). Approximate solution of singular integral equations of the first kind with Cauchy kernel. Appl. Math. Lett. 22:651-657.

Gakhov FD (1966). Boundary Value Problems, Addison-Wesley.

Ladopoulous EG (1987). On the solution of the two-dimensional problem of a plane crack of arbitrary shape in an anisotropic material, J. Eng. Fract. Mech. 28:187-195.

Ladopoulous EG (2000). Singular integral Equations Linear and Non-linear Theory and its Applications in Science and Engineering; Berlin, Springer.

Lifanov IK (1996). Singular Integral Equation and Discrete Vortices. Vso. The Netherlands.

Martin A, Rizzo F (1989). On boundary integral equations for crack problems. Proc. Roy. Soc. A. 421:341-345.

Muskhelishvili NI (1977). Singular Integral Equations. Edited by J. R. M. Radok, Noordhoff International Publishing Leyden.

Prossdorf S (1977). On approximate methods for the solution of one-dimensional singular integral equations, Appl. Anal. 7:259-270.

Prem K, Michael R (2005). Handbook of computational methods for integration. Chapman Hall / CRC Press.

Sheshko M (2003). Singular integral equations with Cauchy and Hilbert kernels and their approximated solutions. The Learned Society of the Catholic University of Lublin, (in Russian).

Zabreyko PP (1975). Integral Equations- A Reference Text, Noordhoff, Leyden, The Netherlands.

Zisis VA, Ladopoulos EG (1989). Singular integral approximations in Hilbert spaces for elastic stress analysis in a circular ring with curvilinear cracks, Indus. Math. 39:113-134.

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Similarities between photosynthesis and the principle of operation of dye-sensitized solar cell

Efurumibe E. L.* and Asiegbu A. D.

Physics Department, College of Natural and Physical Sciences, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria.

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Photosynthesis is the process of converting light energy into chemical energy and storing it in the bonds of sugar; while the operational principle of the dye-sensitized solar cell involve the absorption of light by a dye adsorbed by a photoanode. Electrons are released when the dye is illuminated. The comparison done here showed that there exist a lot of similarities between photosynthesis and the principle of operation of a dye-sensitized solar cell.

Key words: Photosynthesis, dye, solar, cell, similarities.

INTRODUCTION

Dye-sensitized solar cell is a new technology that generates electricity when exposed to sunlight. This technology has opened a new area of research interest for scientists. Currently researchers are working to improve on the photon-to-current conversion efficiency of the dye sensitized solar cell. Some researchers have varied the chemical composition of the dye (Ruthenium complex) with the aim of improving the photo-to-current conversion efficiency (Kuang et al., 2007). It is believed that there exist some similarities between photosynthesis and the operational principle of any dye-sensitized solar cell. Dye-sensitized solar cell, as mentioned above, requires sunlight before it can generate electricity. In the same way, plants require sunlight before they can produce sugar. Before we look into the similarities proper, we need to first look at the various concepts under investigation.

PHOTOSYTHESIS

Photosynthesis is the process of converting light energy to chemical energy and storing it in the bonds of sugar (Pessarakli, 2002). This process occurs in plants and some algae (Kingdom Protista). Plants need only light energy, CO_2, and H_2O to make sugar. The process of photosynthesis takes place in the chloroplasts, specifically using chlorophyll, the green pigment involved in photosynthesis (Farabee, 2007). Photosynthesis takes place primarily in plant leaves, and little to none occurs in stems, etc. The parts of a typical leaf include the upper and lower epidermis, the mesophyll, the vascular bundle(s) (veins), and the stomates. The upper and lower epidermal cells do not have chloroplasts, thus photosynthesis does not occur there. They serve primarily as protection for the rest of the leaf. The stomates are holes which occur primarily in the lower epidermis and are for air exchange: they let CO_2 in and O_2 out. The vascular bundles or veins in a leaf are part of the plant's transportation system, moving water and nutrients around the plant as needed. The mesophyll cells have chloroplasts and this is where photosynthesis occurs (Raghavendra, 2000).

Chlorophyll looks green because it absorbs red and blue light, thus making the red and blue colors invisible to our eyes. It is the green light which is not absorbed that

*Corresponding author. E-mail: efurumibeezinna@yahoo.com

finally reaches our eyes, making chlorophyll appear green. However, it is the energy from the red and blue light that are absorbed and invariably used for photosynthesis. The green light as can be seen is not absorbed by the plant, and thus cannot be used during photosynthesis. The overall chemical reaction involved in photosynthesis is:

$6CO_2 + 6H_2O$ (+ light energy) $\rightarrow C_6H_{12}O_6 + 6O_2$.

This is the source of the O_2 we breathe. It therefore becomes necessary for government at all level to check the act of deforestation going on in the country (Farabee, 2007).

There are two parts to photosynthesis:

1. The light reaction: It happens in the thylakoid membrane and converts light energy to chemical energy (Dahik, 2011). This chemical reaction must therefore need the presence of light before it can occur. Chlorophyll and several other pigments such as beta-carotene are organized in clusters in the thylakoid membrane and are involved in the light reaction. Each of these differently-colored pigments can absorb a slightly different color of light and pass its energy to the central chlorphyll molecule to do photosynthesis (Blankenship, 2002).

2. The dark reaction: It takes place in the stroma within the chloroplast, and converts CO_2 to sugar. This reaction does not need light directly for it to occur, but the products of the light reaction. The dark reaction involves a cycle called the Calvin cycle in which CO_2 and energy from adenosine triphosphate are used to form sugar (Blankenship, 2002).

DYE-SENSITIZED SOLAR CELL

Dye-sensitized solar cells are nanoparticulate photovoltaic cells that generate electricity when exposed to sunlight (Reijnders, 2009). Dye sensitized solar cells offer the prospect of very low-cost fabrication and present a range of attractive qualities that will facilitate market entry (Grätzel and Durrant, 2008). In most cases, the dye-sensitized solar cell is called in conjunction with the word: “mesoscopic” as: dye-sensitized mesoscopic solar cell. The word mesoscopic refers to a small scaled size (usually nanoscaled size). The standard dye-sensitized solar cell that uses Titanium oxide as its anode is often times called: Gratzel cell (Reijnders, 2009). This is because Michael Grätzel was the one who first developed a workable dye-sensitized cell based on Titanium oxide anode and a Rheutenium Dye. Grätzel received a millennium technology prize award for his work in 2010 (Hollister, 2010).

The progress realized recently in the fabrication and characterization of nanocrystalline materials has opened up vast new opportunities for the dye-sensitized solar cells. The Dye sensitized mesoscopic solar cells achieves optical absorption and charge separation processes by the association of a sensitizer (dye) as light-absorbing material with a wide-bandgap semiconductor (TiO_2) of nanocrystalline morphology (O’Regan and Grätzel, 1991). Dye-sensitized solar cells are said to be photo-electrochemical cells that make use of electrolytes in place of semiconductors. A sketch of the mesoscopic dye-sensitized solar cell is given in Figure 1.

In the working of the cell, when the sensitizer dye is illuminated, it releases electrons which are injected by a fast process into the conduction band of the titanium oxide (TiO_2) anode. As can be seen from Figure 2, the sensitizer dye is attached to the surface of the mesoporous titanium oxide nanocrystalline thin film. Photoexcitation of the sensitizer dye results in the injection of an electron into the conduction band of the TiO_2. The injected electron transports through the anode towards the external terminals where they could be utilized by a load. On the other hand, the sensitized dye becomes ionized after release of electrons. This ionized dye is regenerated by electron donation from the electrolyte (Grätzel and Durrant, 2008). The electrolyte on its own is regenerated by the transparent conducting oxide glass coated with platinum. The electrolyte in the Grätzel cell is an iodide (triodide) redox couple dissolved in a liquid organic solvent.

COMPARING PHOTOSYNTHESIS IN PLANT WITH THE OPERATIONAL PRINCIPLE OF THE DYE-SENSITIZED SOLAR CELL

The leaf of a plant represents the dye in the dye-sensitized solar cell. Under illumination, the dye in the dye-sensitized solar cell releases electrons which are transported through the anode to the external terminals for utilization (Figure 1). Similarly, under illumination by sunlight, the leaf of a plant releases sugar and oxygen. The oxygen goes into the atmosphere through the stomates while the sugar ($C_6H_{12}O_6$) is stored in the upper and lower epidermis through the vascular bundles. It can be observed that in both cases the products of the reactions (electrons, oxygen and sugar) are transported to areas of need. The electrons are transported through the anode to the external terminals, while the oxygen and the carbohydrate are transported through the stomates and the vascular bundles respectively. The chemical reaction involved in photosynthesis is given as:

$6CO_2 + 6H_2O$ (+ light energy) $\rightarrow C_6H_{12}O_6 + 6O_2$

while that involve in the operation of dye-sensitized solar cell is given as:

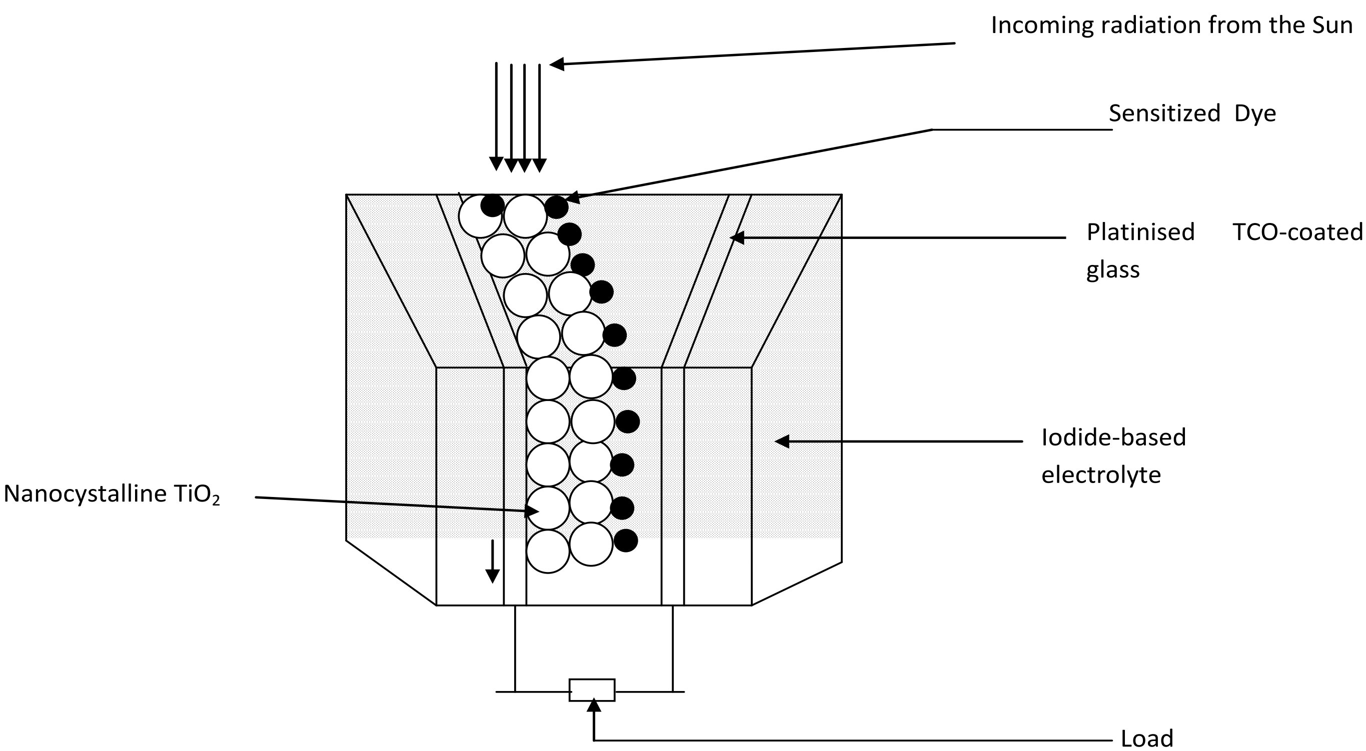

Figure 1. A sketch of the mesoscopic dye-sensitized solar cell.

$D_c + \text{light} \rightarrow D_c^{+} + e^{-}$
$D_c^{+} + 2I \rightarrow D_g + I_2^{-}$
$2I_2^{-} \rightarrow I_3^{-} + I^{-}$

SIMILARITIES

The aim of the study is to deduced the similarities between photosynthesis and the operational principle of dye-sensitized solar cell. Following the study, it was observed that both processes need light from the sun to fuction. In both cases, there is absorbtion of energy from the sun. In dye-sensitized solar cell, the absorbed energy is needed to release of electrons from the dye, whereas in photosynthesis, the absorbed energy is used to generate carbohydrate and oxygen from the leaf. In both cases there is transportation of the products down to areas of needs. In dye-sensitized solar cell, the electrons are transported through the anode to the external terminal where they are utilized, whereas the carbohydrate generated during photosynthesis is transported through the vascular bundles of the plant to the upper and lower epidermis where they are stored for use. The oxygen generated goes into the atmosphere through the stomates. In both cases there are certain chemical reactions that are involved. For photosynthesis, the chemical reaction occurs in the leaf; whereas the reaction for dye-sensitized solar cell occurs at the

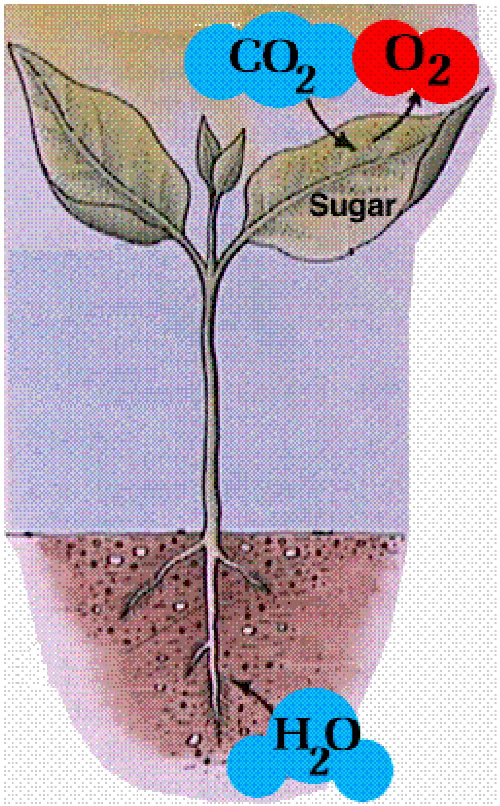

Figure 2. Photosynthetic process.

interface between the dye and the electrolyte. Both photosynthesis and the operational principle of dye sensitized solar cell are environmental friendly, since pollutants are not released in both processes. Lastly there is conversion of energy from light to electricity (in Dye-sensitized solar cell) and from light to chemical (in plants). In essence, in both cases there is conversion of light energy to other forms energy.

CONCLUSION

Having compared photosynthesis in plants and the operational principle of dye-sensitized solar cell and seen that they have similar feature, it is worthy of mentioning that dye-sensitized solar cell is a technology to be embraced. More researches should be carried out in this field of study that is mimicing photosynthetic process in its operation. For when dye-sensitized solar cell is fully adopted, the problem of 'carbon emmission' will be greatly reduced. And by this our environment would be protected.

REFERENCES

Blankenship RE (2002). Molecular Mechanism of Photosynthesis, Wiley-Blackwell, accessed online on 16/07/2011 from: www.books.google.com.

Dahik AC (2011). Investigating the effect of using artificial or direct light on the rate of photosynthesis, accessed online on 16/07/2011 from: www.free-ebookslink.com.

Grätzel M, Durrant RJ (2008). Dye-sensitized Mesoscopic solar cells, Giacomo Ciamician, New York. P. 217.

Farabee MJ (2007). Photosynthesis. Accessed from: http://www2.estrellamountain.edu/faculty/ farabee/ biobk/BioBookPS.html on 07/06/2011.

Hollister S (2010). Michael Grätzel, inventor of the dye-sensitized solar cell, wins 2010 Millennium Technology Prize, accessed on 10/07/2011 from: http://www.engadget.com.

Kuang D, Klein C, Snaith HJ, Humphry-Baker R, Zakeeruddin SM, Grätzel M (2008). A New ion-coordinating Ruthenium Sensitizer for Mesoscopic Dye-sensitized Solar Cells, Inorganica Chimica Acta. 361: 699-706.

O'Regan B, Grätzel M (1991). A low cost, high-efficiency solar cell based on dye-sensitized colloidal TiO_2 films, Nature. 353:737-739.

Pessarakli M (2002). Handbook of Plant and Crop Physiology, accessed online on 16/07/2011 from: www.books.google.com.

Raghavendra AS (2000). Photosynthesis: A Comprehensive Treatise, Cambridge University Press, accessed online on 16/07/2011 from: www.books.google.com.

Reijnders L (2009). Design Issues for Improved Environmental Performance of Dye-sensitized and Organic Nanoparticulate solar cells. J. Cleaner Prod. 18(2010):307-312, published by Elsevier.

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Adsorption study of *Nymphaea alba* for the removal of manganese from industrial waste water

Shazia Khurshid, Shaheen Kausar*, Sadia Afzal, Ahmed Adnan, Arshad Mehmood, Muhammad Arshad and Zafer Ahmad Bukhari

Department of Chemistry, G. C. University Lahore, Pakistan.

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The application of adsorbents obtained from *Nymphaea alba* as a replacement for costly conventional methods of removing heavy metal ions from waste water has been reviewed. Detailed adsorption study of manganese on *N. alba* was investigated. Batch adsorption study was carried out as a function of amount of adsorbent, contact time, thermal treatment, pH and agitation speed. The adsorbent to solution ratio and the metal ion concentration in the solution affects the degree of metal ion removal. Instrumentation employed was atomic absorption spectrometer. *N. alba* was an excellent adsorbent as compared to number of other low cost adsorbents.

Key words: Manganese, low cost adsorbent, industrial waste water, *Nymphaea alba*.

INTRODUCTION

Pollution is the introduction of contaminants into the natural environment that causes adverse change. Water is contaminated by the discharge of industrial waste, untreated domestic sewage into surface waters. Point and non point sources are causes of pollution to our water resources due to tremendous population growth (Franklin, 1991).

Millions of people worldwide are suffering with the shortage of fresh and clean drinking water. Rapid industrialization, population expansion and unplanned urbanization have largely contributed to the severe pollution to water reservoirs and surrounding soils. It is well known that 70-80% of all illnesses in developing countries are related to water contamination. Pollutants discharged in wastewaters are toxic to aquatic life thus disturbing natural bio balance of water body in which the waste is being disposed (Sundarrajan et al., 2000, Ali, 2010).

Heavy metals are the major contaminant in industrial wastewater. The metals commonly include cadmium (Cd), lead (Pb), copper (Cu), zinc (Zn), nickel (Ni) and chromium (Cr). These metals deposit in human bodies either through direct intake or through food chains (Argun and Dursun, 2008). Polluted water can be treated by different remediation techniques, for example ion exchange, oxidation, electrodialysis, reverse osmosis, electrolysis and adsorption. The basic principles of these techniques used for the removal of pollutants are based on their chemical, electrical, physical, biological and thermal properties. Treatment costs of these technologies ranges from 10-450 US dollars per cubic meter of treated water except adsorption technique. The cost of water treatment using adsorption is 5.0-200 US dollars per cubic meter of water (Gupta et al., 2012). Adsorption is considered to be the best wastewater treatment technique due to its universality, ease of handling and inexpensiveness. Adsorption can remove soluble and insoluble pollutants. The removal capacity of this method is up to 99.9%. Therefore adsorption has been used for the removal of a variety of pollutants from different

*Corresponding author. E-mail: missshaheenkausar@gmail.com, Tel: 03338734321.

contaminated water sources (Ali et al., 2012).

Manganese is the part of natural environment. Asyndrome related to manganese chronic exposure to higher levels of manganese may result in the symptoms of lethargy, mask like face, tremors and psychological disturbances. Respiratory effects have also been reported in workers chronically exposed by inhalation (Flynn and Susi, 2009).

Water containing Mn has rust colour, that is the cause of staining of faucets, sinks, or laundry, and has an off-taste or odour. Manganese levels vary in different places in a water body. Infants should not drink water that is above the health advisory level of 300 μg/L (Agency for Toxic Substances and Disease Registry (ATSDR) 1997).

Due to over population and industrialization, demand for water is increasing day by day. So, alternate sources of clean water are needed and wastewater treatment may serve this purpose. Among various water treatment techniques, adsorption on activated carbon is on the top due to its universal nature. Activated carbon is the best adsorbent able to adsorb inorganic as well as organic pollutants that contaminate water resources. In addition to activated carbon, other adsorbents can also be used to remove toxic metal ion from waste water (Ali, 2010). Dead biomasses of different origin constitute cheap adsorbent for inorganic pollutants as well as organic contaminants in the industrial waste water (Panayotova and Velikov, 2008; Pesavento, 2003).

Plant wastes are inexpensive as they have no or very low economic value. Most of the adsorption studies have been focused on untreated plant wastes such as papaya wood, maize leaf, tea leaf powder, lalang leaf powder, rubber leaf powder, peanut hull pellets, sago waste, salt bush leaves tree fern, rice husk ash and Neem bark, grape stalk wastes, etc. Some of the advantages of using plant wastes for wastewater treatment include simplest of the technique, little processing, good adsorption capacity, selective adsorption of heavy metal ions, low cost, free availability and easy regeneration (Burkel and Stoll, 1999).

Nilofar flower grows abundantly near or in water bodies in normal climatic conditions of Pakistan (Figure 1). *Nymphaea alba* is large, cup-shaped, white flowers up to 20 cm across. *N. alba* (Nilofar flower) is easily available in Pakistan. It is present in ponds and lakes. The present paper describes the use of flowers of *N. alba* as an adsorbent from industrial waste effluents. It is highly economic, naturally occurring and effective adsorbent. It can be used not only for manganese but for other heavy metals like copper, cadmium, chromium, nickel, lead, iron, cobalt, zinc and mercury.

Figure 1. Nilofar flower in water bodies in normal climatic conditions of Pakistan.

METHODOLOGY

All the chemicals and solvents used in experimental work were highly purified and of analytical grade (Merck). Double distilled water was used for preparation and dilution of solutions. Atomic Absorption Spectrophotometer (Shimadzu, Japan AA7000F) was used for the determination of manganese contents. Fuel was acetylene and oxidant was air with the ratio of 1:3, pH meter was used to check pH ranges of solutions, Technico10S-207 Incubator Shaker was used for continuous shaking, Gallenkampe oven was used for heating purpose, Distillation assembly with water vacuum pump.

Preparation of adsorbent

N. alba from different ponds was washed with double distilled water. First it was dried in sun and then in oven at 50°C for one hour. After complete drying it was powdered using a mechanical grinder, sieved and stored to save it from moisture and contamination.

Adsorption experiment

Waste water samples were collected in plastic bottles from different industries and were refrigerated. The samples were doubly filtered before the treatment. Dilutions were made for each sample by diluting 10 ml of each sample to 100 ml. 50 ml of these effluents were transferred to different conical flasks. Atomic absorption spectrophotometer was used for the determination of Mn. Chemical compounds were ionized into free analyte by burners that provide the heat energy. Standard solutions of different concentrations were prepared and 50 ml from each solution was transferred to different conical flasks. Solutions were analysed for concentration of metals after filtration.

Various parameters that were used to check the adsorption behaviour of *N. alba* were:

1. Effect of concentration of adsorbent.
2. Effect of contact time.
3. Effect of temperature.
4. Effect of pH.
5. Effect of agitation speed.

A small portion of waste water was filtered off through membrane filter (0.45 μm).Filtered samples (50 ml) were transferred in Erlenmeyer conical flask. Total concentration of manganese in the samples was determined by means of atomic absorption spectrometer. Adsorption experiments were made at room temperature after using 50 ml of known concentration of

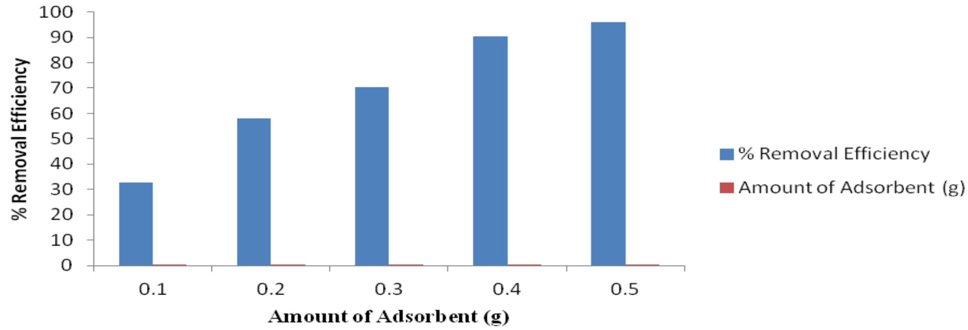

Figure 2. Effect of amount of adsorbent on adsorption of Mn.

manganese solutions in different flasks. Various amounts of adsorbent were put in each flask and these were shaken at 100 rpm Biosorption was monitored by measuring the decline in concentration of Mn in the samples by atomic absorption spectrometer. The effect of various other parameters on adsorption was also studied.

The effect of concentration of adsorbent

50 ml of 50 ppm solutions were taken in conical flasks. Different amounts of adsorbent (0.1 g, 0.2 g up to 0.7 g) were added in the solutions and were agitated at a speed of 100 rpm for 30 min. The solutions were double filtered and analysed by atomic absorption spectrophotometer.

The effect of contact time

50 ml of 50 ppm solutions were taken in conical flasks. 0.5 g of adsorbent was added in each flask and was shaken at 150 rpm for various time intervals (10, 20, 30, 40 up to 100 min). Then solutions were doubly filtered off and were analysed by atomic absorption spectrophotometer.

The effect of thermal treatment on Mn adsorption

50 ml of 50 ppm solution was taken in conical flasks. 0.5 g of adsorbent was added to the flasks. Flasks were agitated at 150 rpm for 100 min at various temperature ranges (10, 20 to 90°C). Solutions were filtered and analysed by atomic absorption spectrophotometer.

The effect of pH on Mn adsorption

50 ml of 50 ppm solutions were taken in conical flasks and pH of flasks was adjusted over a pH range of 1-10 by means of 0.1M NaOH/HCl solutions. 0.5 g adsorbent was added in each flask. Flasks were shaken in shaker at 150 rpm for 100 min at 70°C. Solutions were double filtered and analysed by atomic absorption spectrophotometer.

The effect of agitation speed on Mn adsorption

Adsorbent (0.5 g) was added in 50 ml of 50 ppm solution and pH was adjusted at 7. Flasks were agitated at 70°C for 100 min at various agitation speeds as 25, 50, and 75, 100 up to 225 rpm. Solutions were double filtered and analysed by atomic absorption spectrophotometer.

Removal of manganese from chemical industry

Waste effluent collected from a chemical industry near Muridke (District Sheikhpura) was treated using same method as used for manganese standard solutions. 50 ppm of diluted waste water was double filtered. Its concentration was measured by atomic absorption spectrometer.

RESULTS AND DISCUSSION

Removal of manganese (II) from industrial waste water through *N. alba* (Nilofar flower) is an efficient technique. *N. alba* is locally present in lakes and ponds in Pakistan. The use of *N. alba* for the removal of toxic heavy metal like copper, cadmium, chromium and manganese from different industrial effluent is very easy, efficient and adoptable method. Method adopted was to find the initial concentration of Manganese (II) in the solutions and then determining the concentration after treatment. The difference showed the removal of manganese metal from waste water.

Effect of various parameters on adsorption of manganese

Effect of amount of adsorbent

A 50 ml aliquot of sample solution containing 24.26 ppm Manganese (II) was transferred to each flask after filtration. Different amounts of adsorbent ranging from 0.2 to 0.7 g were added to 50 ml waste water. Solutions were shaken at 175 rpm for 100 min at 60°C. Samples were analysed by atomic absorption spectrometer. With varying amounts of the adsorbent, adsorption varied and the maximum adsorption of 96% was attained with 0.5 g of adsorbent per flask (Figure 2).

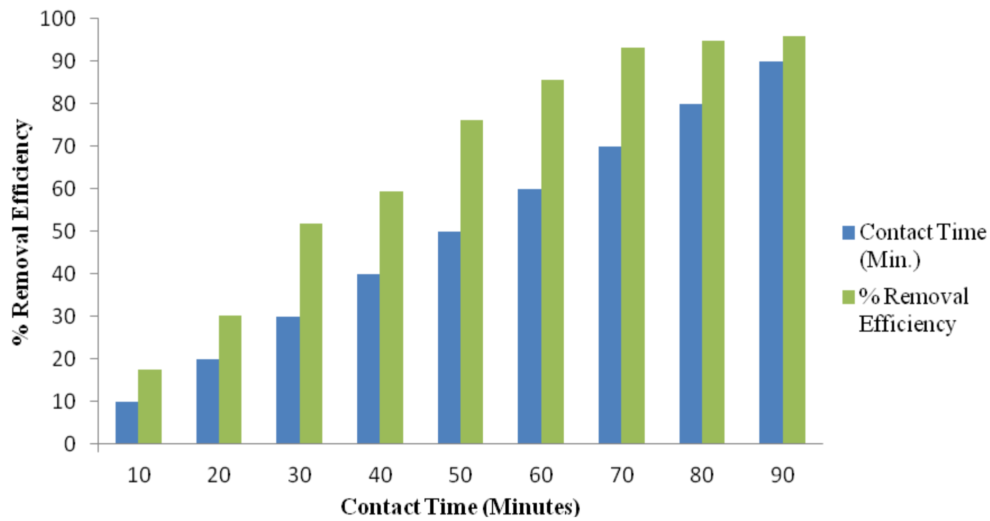

Figure 3. Effect of amount of contact time on % removal of Mn.

The effect of contact time on manganese adsorption

Effect of length of contact between the adsorbent and the solution was studied by keeping other parameters constant. Time duration varied from 20 to 140 min. Results revealed that more contact time resulted in better removal of metal ions from solution. The maximum removal of 96% was observed when contact time of 100 min after which there was insignificant change (Figure 3).

Effect of temperature on manganese adsorption

The effect of temperature was investigated using a temperature range of 10 to 90°C keeping all other parameters constant. To counter the evaporation at elevated temperatures (above 40°C), deionized water was added into flasks at regular short intervals to keep uniform volume through the experiment. Adsorption of Manganese (II) ions from waste water increased with increasing temperature and the maximum removal of 96% was observed at 70°C. A further increase in temperature did not enhance the adsorption (Figure 4).

Effect of pH on manganese adsorption

Waste water samples were treated at various pH ranges like 1 to 10. Maximum removal of 96% was seen at pH 7 (Figure 5).

Effect of agitation speed on adsorption of manganese

By shaking adsorbent and metal ions mix well and have more contact to interact. Waste water samples were shaken in incubator shaker at various speeds and it was found that the speed of adsorption increased by increasing the shaking and reached the maximum value at agitation speed of 150 rpm. At this speed, the maximum adsorption of 96% was observed (Figure 6 and Table 1).

Conclusion

N. alba can act as good adsorbent not only for the removal of manganese ions but also for various other toxic metals like lead, mercury, copper, chromium, cadmium, cobalt, nickel and iron. The removal of manganese (II) from industrial waste water through *N. alba* (Nilofar flower) was found to be an efficient technique. It was an efficient strategy and its performance was the best at temperature of 70°C and pH 7.0. The technique employed was very simple and environment friendly as no manmade chemicals were involved. This method can be adopted by various industries for the waste water treatment. The experimental parameters at temperature of 70°C, pH 7 and minimum amount of adsorbent level of 1.0% were considered to be 96% efficient in removing manganese (II) from industrial waste water. So industries can apply this methodology for removal of toxic metals from industrial waste water effluents, thus preventing land and water pollution.

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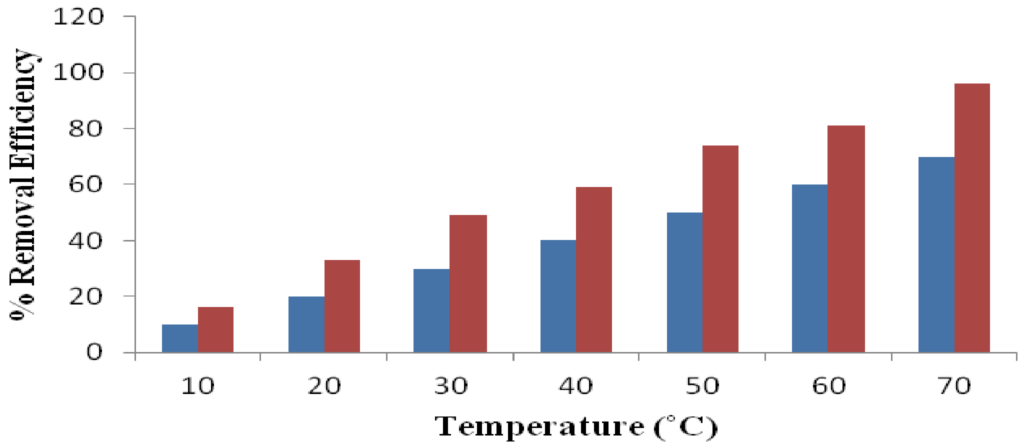

Figure 4. Effect of temperature on % removal efficiency of Mn.

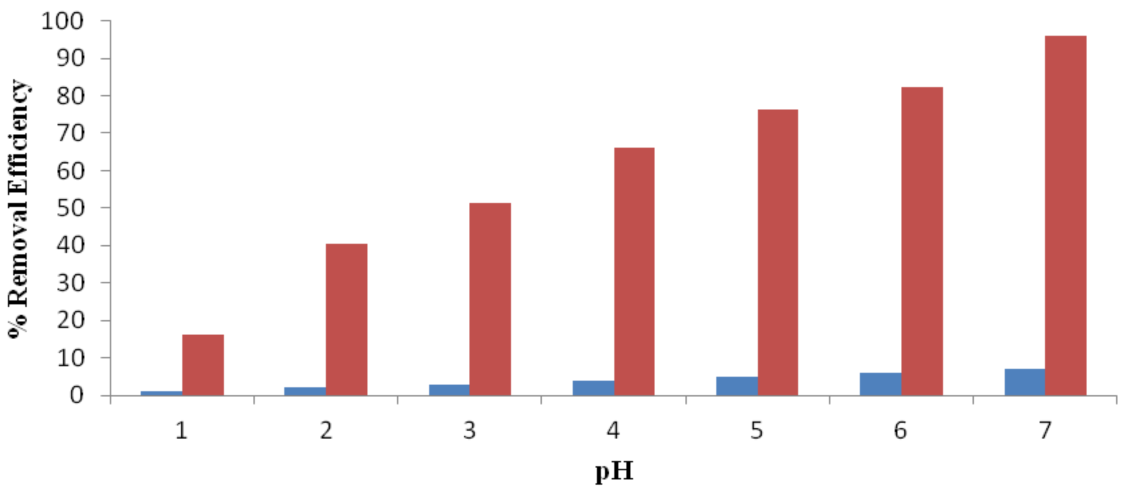

Figure 5. Effect of pH on removal of Mn.

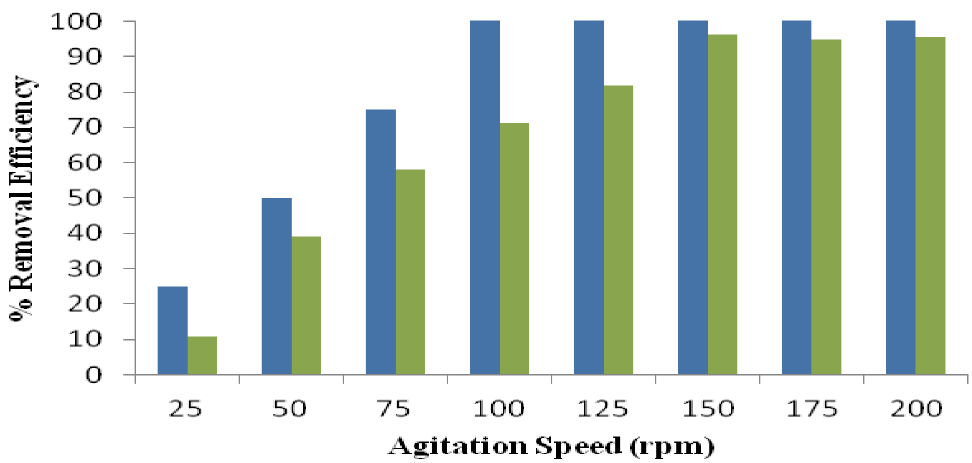

Figure 5. Effect of agitation speed on the removal of Mn.

Table 1. Concentration of manganese in different industrial waste water.

Industries	Concentration of Manganese (ppm.)
I	0.019
II	0.135
III	0.322
IV	5.445
V	18.325
VI	19.464
VII	0.039
VIII	24.268
IX	0.468
X	0.623

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REFERENCES

Agency for Toxic Substances and Disease Registry (ATSDR). (1997) Toxicological Profile for Manganese. U.S. Department of Health and Human Services, Atlanta, GA.

Ali I, Mohd A, Tabrez AK (2012). Low cost adsorbents for the removal of organic pollutants from waste water. J. Environ. Manage. 113:170-183.

Ali I (2010). The quest for active carbon adsorbents substitutes: Inexpensive adsorbent for toxic metal ions removal from wastewater, separation and purification reviews, 39:95-171.

Argun ME, Dursun S (2008). A new approach to modification of natural adsorbent for heavy metal adsorption. Bioresour. Technol. 99:2516-2527.

Burkel RS, Stoll RC (1999). Naturally occurring arsenic in sandstone aquifer water supply wells of North Eastern Wisconsin. Ground Water Monit. Remed.19:114-121.

Flynn MR, Susi P (2009). Neurological risks associated with manganese exposure from welding operations. Int. J. Hyg. Environ. Heal. 212:459-469.

Franklin LB (1991). Low cost adsorbents for the removal of organic pollutants from wastewater. Waste water engineering: treatment disposal and reuse.

Gupta VK, Ali I, Saleh TA, Nayak A, Agarwal S (2012). Chemical treatment technologies for waste-water recycling, *RSC.* 2:6380-6388.

Panayotova M, Velikov B (2008). Influence of zeolite transformation in a homoionic form on the removal of some heavy metal ions fromwastewater. J. Environ. Sci. Health. 38:545-554.

Pesavento M (2003). Adsorption of lead (II) and copper (II) on activated carbon by complexation with surface functional groups. Anal. Chem. Acta. 480:171-180.

Sundarrajan M, Fernandis AZ, Subrahmanyam G, Prabhudesai S, Krishnamurthy S, Rao K (2000). Over expression of G1/S cyclins and PCNA and their relationship to tyrosine phosphorylation and dephosphorylation during tumour promotion by metanil yellow and malachite green. Toxicol. Lett.116:119-130.


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